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When frictions favour
information revelation

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Abstract

We study information revelation in markets with pairwise meetings. First, we reconsider the one-sided case within constant entry flow model. The same question has been studied in an identical framework in Serrano and Yosha (1993). We prove that there exists an additional equilibrium not detected by Serrano and Yosha (1993). We show that this equilibrium is characterized by incomplete information revelation. Until now, no equilibrium with incomplete revelation of information was known in this model. Our second main result is that, at this new equilibrium, information revelation is worse when frictions are weaker. One prove also that increasing the frictions is a Pareto improvement. Finally, we show that those properties should also characterize some equilibria of the two-sided case studied by Wolinsky (1990).

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Introduction

In the general equilibrium approach of competitive markets, trade is centralized: all sellers and buyers of a particular commodity meet in the same location, and all trade takes place simultaneously at the same price. A central mechanism, often presented as an auctioneer, is needed to choose the price which equates demand with supply. Moreover, no trade takes place until the price has been made public. Frictions are completely absent of the model since nothing creates an obstacle to trade. By modifying this setup, one can introduce asymmetric information about the value of the good being traded, i.e. there is uncertainty about common values. Analyzing information revelation in this context leads to the conclusion that unfettered market prices aggregate and reveal all relevant information that is dispersed asymmetrically among agents - so that prices allow the final allocation of resources to be efficient.¹

Some assumptions as well as certain conclusions, are difficult to accept as obvious. Indeed, it is quite natural to be reluctant to endorse the extreme centralization or the conclusion of full information revelation which seems to be battered by facts at least in numerous situations if not all. So, we can be interested by the literature begun by Wolinsky (1990). In that approach, the main goal is to study the information revelation and efficiency properties of a decentralized market, where there is no auctioneer and transactions take place via pairwise meetings of agents.

In his seminal paper, Wolinsky (1990) addressed the following qualitative question *to what extent is the information revealed to uninformed agents through the trading process, when the market is in some sense frictionless?* More precisely, *does the decentralized process give rise to full revelation results as derived by the literature on rational expectations for centralized and competitive environments?* The main message was rather pessimistic since it turned out that the information is not fully revealed to uninformed agents, even when the market is in some sense approximately frictionless.

Wolinsky (1990) does not imply anything concerning the impact of possible frictions on the efficiency of the market. In its current state, the literature is silent on the issue of the second best : *When frictions are unavoidable;*

¹The rational expectations equilibrium (REE) is an equilibrium concept used in the GE framework. In a centralized analogue of the case we study, it exhibits full information revelation and is an interim incentive efficient mechanism, a sort of *First Welfare Theorem*. This efficiency concept is presented in Holmstrom and Myerson (1983). A definition of REE and a discussion of its information-revelation properties can be found in Radner (1982).

will the lowest frictions imply the most complete information revelation ?

In order to address this question, we consider an equilibrium with incomplete revelation of information in the model of Serrano and Yosha (1993). Their model is very close to the one of Wolinsky (1990) but slightly less complicated. Both models are built on a market with sellers on one side and buyers on the other one. Wolinsky (1990) analyses a situation where uninformed agents belong to both sides of the market while only some buyers are uninformed in the case studied by Serrano and Yosha (1993).

Our main result is that frictions favour information revelation. One could however object that what matters is not information revelation but the welfare and we hence analyze the issue from a welfarist point of view. Actually, the result is rather reinforced than weakened by the modification of perspective. Not only does a rise of the frictions increase the total welfare, but it is even a Pareto improvement.

Readers familiar with the literature could be surprised by the fact that we speak about an equilibrium with incomplete information revelation in the one-sided case. The construction of this equilibrium is also a substantial part of our contribution. On one hand, it is not so surprising since the existence of this kind of equilibrium was not completely excluded by Serrano and Yosha (1993). On the other hand, until now it was considered as certain that, for the parameters values that we use, no equilibrium could imply incomplete revelation of information.

In order to generalize our result, we prove that an equivalent equilibrium exists in the two-sided case. So, the idea that frictions favour information revelation extends at least to some equilibria of the more general case that Wolinsky (1990) considers.

While the message of Wolinsky (1990) is pessimistic: no equilibrium implies complete information revelation when the frictions disappear; the one-sided case, even with our contribution, brings a conclusion more optimistic since there exist equilibria characterized by complete information revelation when the market becomes approximately frictionless. This difference in term of results between Serrano and Yosha (1993) and Wolinsky (1990) is usually explained by a noise force created by the presence of uninformed agents on both sides of the market in Wolinsky (1990). Our results do not challenge completely this view but modify nevertheless the mechanism and the necessity of noise force to prevent complete information revelation. So, our results contribute also to improving our understanding of the differences between the one- and two-sided cases.

In the first section, we present the model. The second section provides some characterizations of the equilibria that are useful in the following sec-

tions. The third section introduces the new equilibrium. The properties of this equilibrium are analyzed in section four. The fifth section presents the equivalent equilibrium in the two-sided case. The noise force is discussed in section six. In the last section, we conclude.

1 The Model

We consider the model of Serrano and Yosha (1993) and study it without assuming an *a priori* stationarity of the equilibrium.

Time runs discretely from 1 to ∞ . Each period is identical. On one side, there are sellers who have one unit of an indivisible good to sell. On the other side, there are buyers who want to buy one unit of this good. In each period, a continuum of measure M of new sellers and the same quantity of buyers enter the market. The agents quit the market when they have traded. Hence, the number of sellers is always equal to the number of buyers.

There exist two possible states of the world, which influence the payoff of the agents. If the state is low (L), the cost of production (c_L) for the sellers but also the utility (u_L) of the buyers are low. If the state is high (H), the corresponding parameters (c_H and u_H) are high. The state remains identical during all the periods.

All sellers know the state of the world, whereas not all of the buyers are perfectly informed. Among the newcomers, there is a fraction x_B of buyers who are perfectly informed. The remaining buyers are uninformed and possess a common prior belief $\alpha_H \in [0, 1]$ that the state is H and $(1 - \alpha_H)$ that the state is L .

At each period, all the agents are randomly matched with an agent of the other type.² At each meeting, the agents can announce one of two prices : p^H and p^L . If both agents announce the same price, trade occurs at this price. If a seller announces a lower price, trade occurs at an intermediate price p^M . If a seller announces a higher price, trade does not occur. The different parameters are assumed to be ordered such that :

$$c_L < p^L < u_L < p^M < c_H < p^H < u_H \quad (1)$$

Remaining on the market implies a zero payoff. The instantaneous payoff when a transaction occurs is the price minus the cost for a seller and the utility minus the price for a buyer. All agents discount the future by a constant factor δ .

²See Duffie and Sun (2007) for a rigorous proof of the existence of independent random matching between two continua.

In state H , we call p^H the *good* price because trade at other prices implies a loss for the sellers. Similarly, the price p^L is the *good* price in state L because trade at other prices involves loss for the buyers.

After each meeting with a seller who announces p^H , a buyer will update his belief α_H according to Bayes' rule. If an uninformed buyer meets a seller who announces p^L , he will know that the state of the world is L , but it does not really matter any more, since this buyer will trade and leave the market.

It is convenient to say that a seller (resp. a buyer) plays *soft* when he announces p^L (resp. p^H) and *tough* when he announces p^H (resp. p^L). When an agent plays *soft*, he is ensured to trade and to quit the market. Hence, to describe completely the strategy of an agent, it is sufficient to give the number of periods in which he plays *tough*. The strategy of an agent might depend on the period of entry on the market. We note $n_{SH}(t)$ the number of periods during which a seller plays *tough* when he enters at time t on a market which is in state H . Similarly, we define $n_{SL}(t)$, $n_{BH}(t)$, $n_{BL}(t)$. Finally, we define $n_B(t)$ as the strategy of an uninformed buyer, which is independent of the state of the world.

An equilibrium is a profile of strategies where each agent is maximizing his expected payoff, given the strategies of the other agents. All parameters $(p^H, p^M, p^L, c_H, c_L, u_H, u_L, x_B, \delta, \alpha_H)$ are common knowledge.

We define now the proportions of agents who play *tough* when state is L . The proportion of the total number of buyers in the market who at period t announce p^L is called B_L^l . Similarly, S_L^h is the proportion of sellers who at period t announce p^H . These values are known to all agents. Naturally, B_H^l and S_H^h are the equivalent proportions when the world is in state H . Let K_H and K_L be the total number of sellers (and therefore for buyers) in the market in state H and in state L . We are at a steady state when K_H , K_L and the four proportions - B_L^l , B_H^l , S_L^h and S_H^h - are constant.

2 Characterization of the Equilibria

In the following claim, we characterize the equilibrium strategies of informed buyers and of sellers in state H .

Claim 1 *In any equilibrium $n_{SH} = \infty$, $n_{BL} = \infty$ and $n_{BH} = 0$.*

Proof An informed seller in state H knows that his payoff will be negative if he trades at an other price than p^H . Since the payoff of perpetual disagreement is 0, he will always prefer to play *tough* even if it implies a long delay before trading. The reasoning is identical for an informed buyer

in state L . An informed buyer in state H will understand that $n_{SH} = \infty$ and thus he will never trade while he plays *tough*. Playing *tough* only delays the payoff. So, it is better for this kind of buyer to play immediately *soft*.

Concerning notations, $V_{SL}(n; B_L^l)$ indicates the expected payoff for a seller in state L when he plays *tough* during n periods. By definition, the optimal strategy for an agent maximizes its expected payoff. So, an equilibrium has to satisfy

$$n_{SL} \in \arg \max_n V_{SL}(n; B_L^l) \quad (2)$$

Similarly, we denote $V_B(n; \alpha_H; S_L^h; S_H^h)$ the expected payoff for an uninformed buyer and the following condition has to be satisfied at any equilibrium :

$$n_B \in \arg \max_n V_B(n; \alpha_H; S_L^h; S_H^h) \quad (3)$$

The focus on steady-state imposes the following restrictions:

$$M = K_H(1 - S_H^h B_H^l) \quad (4)$$

$$M = K_L(1 - S_L^h B_L^l) \quad (5)$$

$$K_L(1 - B_L^l) = M[x_B(S_L^h)^{n_{BL}} + (1 - x_B)(S_L^h)^{n_B}] \quad (6)$$

$$K_H(1 - S_H^h) = M(B_H^l)^{n_{SH}} \quad (7)$$

$$K_H(1 - S_L^h) = M(B_L^l)^{n_{SL}} \quad (8)$$

$$B_H^l = \frac{(1 - x_B)n_B}{(1 - x_B)(n_B + 1) + x_B} \quad (9)$$

The two first equations are the steady state conditions for the market size in the two states of the world. M is the number of entering buyers (resp. sellers). This number has to be equal to the number of exiting buyers (resp. sellers). This last number is equal to the total amount of buyers (resp. sellers) in the market (K_H or K_L according to the state of the world) multiplied by the probability of reaching an agreement. The unique possibility for a match to end on a disagreement is when the meeting happens between two *tough* agents. So, the probability of disagreement in state H (resp. in state L) is given by $S_H^h B_H^l$ (resp. $S_L^h B_L^l$). The probability to reach an agreement is then given by $1 - S_H^h B_H^l$ (resp. $1 - S_L^h B_L^l$).

Equations (6), (7) and (8) are the conditions which ensure the stationarity for the proportions of *tough* buyers in state L , and of *tough* sellers in the two states. The left-hand side is the number of agents who play *soft*. The

right-hand side is the number of buyers who have switched during this period to playing *soft*. This is also the total number of buyers who are playing *soft* because any buyer who plays *soft* in the previous period has for sure transacted and has left the market.

We do not write the stationarity condition for the proportion of *tough* buyers in state H . Actually, this condition is identical to (4). The explanation is the following one. Since sellers are all *tough*, trade - and therefore exit - occurs whenever a buyer switches to *soft*. Thus, the number of buyers who play *soft* in a given period is also the number of buyers who trade and exit the market in this period.

We need an additional equation to complete the system which is equation (9). In state H , since all the sellers are playing *tough*, a buyer who plays *tough* does not trade. Therefore, all the uninformed buyers who entered the market $n_B - 1$ periods ago or later are still in the market and are playing *tough*. Their number is $M(1 - x_B)n_B$. The uninformed buyers who entered the market n_B periods ago are also in the market but are playing *soft*. Their number is $M(1 - x_B)$. Uninformed buyers who entered previously have already left the market. Thus, there is no more uninformed buyers in the market. Informed buyers remain in the market for exactly one period. Therefore the total number of buyers in the market is $M(1 - x_B) + M(1 - x_B)n_B + Mx_B$, and the fraction of *tough* buyers is as described in (9).

3 The Elusive Equilibrium

In a first step, we assume that $n_B = 0$ and $S_L^h < 1$. Hence, the characterization, established in the previous section, takes a particular form given these assumptions ($n_B = 0$ and $S_L^h < 1$).

The second step is devoted to the determination of n_{SL} , the best strategy of a seller in state L .

Then we have to check in a third step that $n_B = 0$ is indeed the best response for an uninformed buyer.

First Step

Let us write the particular form of the characterization when $n_B = 0$ and $S_L^h < 1$.

$$M = K_H \quad (10)$$

$$M = K_L(1 - S_L^h B_L^l) \quad (11)$$

$$S_L^h = \frac{(B_L^l - x_B)}{(1 - x_B)B_L^l} \quad (12)$$

$$S_H^h = 1 \quad (13)$$

$$S_L^h = 1 - (B_L^l)^{n_{SL}} \quad (14)$$

$$B_H^l = 0 \quad (15)$$

$$n_{SL} \in \arg \max_n V_{SL}(n; B_L^l) \quad (16)$$

$$n_B = 0 \quad (17)$$

$$n_{SH} = +\infty \quad (18)$$

$$n_{BL} = +\infty \quad (19)$$

$$n_{BH} = 0 \quad (20)$$

Second Step

This step is devoted to determining n_{SL} , the best strategy of a seller in state L . In that order, we define $\Delta V_{SL}(B_L^l)$ which is the difference in term of gains between playing *soft* tomorrow and playing *soft* today for a seller in state L .

$$\begin{aligned} & \Delta V_{SL}(B_L^l) \\ &= (1 - B_L^l)(p^H - c_L) \\ &+ B_L^l \delta [(1 - B_L^l)(p^M - c_L) + B_L^l(p^L - c_L)] \\ &- [((1 - B_L^l)(p^M - c_L) + B_L^l(p^L - c_L))] \\ &= B_L^l \left[(-p^H + p^M - p^L + c_L) + \delta(p^M - c_L) \right. \\ &\quad \left. + \delta B_L^l(p^L - p^M) \right] + (p^H - p^M) \\ &\equiv B_L^l[X - B_L^l Y] + Z \end{aligned} \quad (21)$$

In the first equality, the two first lines correspond to playing *tough* today and *soft* tomorrow while the third one corresponds to playing *soft* today. If a seller plays *soft* today, he has a probability $(1 - B_L^l)$ to meet a *soft* buyer and consequently to obtain a payoff $(p^M - c_L)$, otherwise (i.e. with probability B_L^l) he will get $(p^L - c_L)$ due to a meeting with a *tough* buyer.

If a seller announces p^H , he will reach an agreement only if he is matched with a *soft* buyer. It occurs with a probability $(1 - B_L^l)$ and the payoff is then $(p^H - c_L)$. Otherwise, with a probability B_L^l , he will remain in the market. In the next period, if he plays *soft*, he has an expected payoff equal to the expression between brackets which must be multiplied by the discount factor δ because trade occurs one period later.

For further use, we denote $B_L^l(\delta)$ the root of $\Delta V_{SL}(B_L^l)$. Using (12), we can also define $S_L^h(\delta) = \frac{(B_L^l(\delta) - x_B)}{(1 - x_B)B_L^l(\delta)}$. It will be later useful to know that $\lim_{\delta \rightarrow 1} B_L^l(\delta) = 1$. It follows from continuity of $B_L^l(\delta)$ and the fact that $B(1) = 1$. It will also be useful to compute $\lim_{\delta \rightarrow 1} S_L^h(\delta) = 1$.

When $\Delta V_{SL}(B_L^l) > 0$, it means that a seller who thinks of playing *soft* today would get a better expected payoff by playing *tough* at least one period more. Since it is true for all the sellers at any point of time, it implies that all the sellers will play *tough*. On the contrary, if $\Delta V_{SL}(B_L^l) < 0$ then all the sellers will play *soft*. Finally, $\Delta V_{SL}(B_L^l) = 0$ leads the sellers to be indifferent between all the strategies, and the proportion of *tough* sellers may a priori take any value. So, optimal strategies are such that

$$\Delta V_{SL}(B_L^l) > 0 \implies S_L^h = 1 \quad (22)$$

$$\Delta V_{SL}(B_L^l) < 0 \implies S_L^h = 0 \quad (23)$$

$$\Delta V_{SL}(B_L^l) = 0 \implies S_L^h \in [0, 1] \quad (24)$$

We have previously assumed $S_L^h < 1$, so we do not consider $S_L^h = 1$. If $S_L^h = 0$ then $n_B = 0$ cannot be the best response when the frictions disappear. Indeed, $n_B = 0$ implies an instantaneous loss if the state of the world is L while $n_B = 1$ ensures a trade with a positive instantaneous payoff, the unique *negative* term comes from the cost of delay which disappears at the same time than the frictions. One can check more formally that $n_B = 0$ is not a best response when $S_L^h = 0$ by observing that ΔV_B introduced later is positive if the proportion of *tough* sellers is nil in state L . So, if it exists, our equilibrium is such that $\Delta V_{SL}(B_L^l) = 0$. In other words : $B_L^l = B_L^l(\delta)$ and $S_L^h = S_L^h(\delta)$ at the equilibrium.

We are interested by the strategy n_{SL} . Individually, the sellers are indifferent between all the strategies but equation (14) has to be satisfied with $B_L^l = B_L^l(\delta)$ and $S_L^h = S_L^h(\delta)$. If this equation gives us an integer for n_{SL} , there is no additional complication. Otherwise, one can say without loss of generality that a proportion r_S chooses $n_{SL} = +\infty$ while the remaining part $1 - r_S$ adopts $n_{SL} = 0$. The amount of *soft* sellers in the market is given by $K_H(1 - S_L^h)$. $M(1 - r_S)$ is the number of sellers who have switched

during this period to playing *soft*. This is also the total number of sellers who are playing *soft* because any seller who plays *soft* in the previous period has for sure transacted and has left the market. Hence, we have $K_H(1 - S_L^h) = M(1 - r_S)$ to replace equation (14). We simplify easily this expression as $S_L^h = r_S$.

Third Step

We started this section by assuming $n_B = 0$ and $S_L^h < 1$. In the second step, we determined n_{SL} according to the initial assumption. The strategy n_{SL} that we found ensures $S_L^h < 1$. It remains to be proven that $n_B = 0$ is indeed the best strategy for an uninformed buyer. In that order and similarly to the previous step, we define $\Delta V_B(S_L^h)$ as the difference of expected payoff between $n_B = 1$ and $n_B = 0$.

$$\begin{aligned}
& \Delta V_B(S_L^h) \\
&= \alpha_H(u_H - p^H)\delta \\
&+ (1 - \alpha_H)(1 - S_L^h)(u_L - p^L) \\
&+ (1 - \alpha_H)\delta S_L^h[(u_L - p^M) + S_L^h(p^M - p^H)] \\
&- \alpha_H(u_H - p^H) - (1 - \alpha_H)[(u_L - p^M) + S_L^h(p^M - p^H)]
\end{aligned}$$

The last line corresponds to the payoff obtained when playing *soft* today. The payoff in state H which is equal to $(u_H - p^H)$ is multiplied by the probability that the state is H . The term in brackets, which is multiplied by the probability that the state is L , is evidently the payoff in state L . This payoff can be written as $(1 - S_L^h)(u_L - p^M)$ (i.e. the probability to meet a *soft* seller times the payoff involved by this meeting) plus $S_L^h(u_L - p^H)$ (i.e. the probability to meet a *tough* seller times the payoff involved). The three first lines correspond to playing *tough* today and *soft* tomorrow. The meaning of the first line is obvious. It is just important not to forget the discount factor δ . Indeed, if the state is H , a buyer who announces p^L does not trade. In the case where the state is L , there is a probability $(1 - S_L^h)$ that a buyer meets a *soft* seller and obtains today $(u_L - p^L)$. If a buyer does not have this luck, which happens with probability S_L^h , he will have tomorrow an expected payoff equal to the expression in brackets. Once again, we must not forget the discount factor.

Obviously, $\Delta V_B \leq 0$ means that $n_B = 0$ is better strategy than $n_B = 1$. What about the other possible strategies? All the differences in expected payoff between $n_B = k + 1$ and $n_B = k$ are smaller than $\Delta V_B(S_L^h)$. Indeed, they will take the same form than $\Delta V_B(S_L^h)$ but with α_H replaced by its

updated value. Note that the expression is decreasing in α_H while due to the Bayes' rule used, the updating process of α_H is increasing. Finally, we get that $\Delta V_B \leq 0$ implies that $n_B = 0$ is the best response for an uninformed buyer.

Hence, we will check that we have effectively $\Delta V_B \leq 0$ at the equilibrium proposed. First, it is easy to check that $\lim_{\delta \rightarrow 1} \Delta V_B(S_L^h(\delta)) = 0$. Then, one can compute its derivative with respect to δ . If this derivative is positive at $\delta = 1$, it means that $\exists \bar{\delta}$ such that $\Delta V_B(S_L^h(\delta)) < 0$ for all $\delta \in [\bar{\delta}, 1[$.

$$\begin{aligned}
\frac{d\Delta V_B(S_L^h(\delta))}{d\delta} &= \alpha_H(u_H - p^H) - (1 - \alpha_H)(p^M - u_L)S_L^h(\delta) \\
&- (1 - \alpha_H)(p^H - p^M)(S_L^h(\delta))^2 \\
&+ (1 - \alpha_H)[p^H - p^M + p^L - u_L - \delta(p^M - u_L)]\frac{dS_L^h(\delta)}{d\delta} \\
&- 2(1 - \alpha_H)(p^H - p^M)\delta S_L^h(\delta)\frac{dS_L^h(\delta)}{d\delta} \quad (25)
\end{aligned}$$

The derivative of S is given by

$$\frac{dS_L^h(\delta)}{d\delta} = \frac{dB_L^l(\delta)}{d\delta} \frac{x_B}{(1 - x_B)(B_L^l(\delta))^2} \quad (26)$$

while the derivative of $B_L^l(\delta)$ evaluated at $\delta = 1$ is equal to

$$\frac{(p^H - p^L)(p^M - c_L) - (p^H - p^M)(p^L - c_L) + (p^H - c_L)(p^M - p^L)}{2(p^M - p^L)(p^H - p^L)} - 1 \quad (27)$$

Finally, we obtain the condition that ensures the positivity of $\frac{d\Delta V_B(S_L^h(\delta))}{d\delta}$ at $\delta = 1$:

$$\alpha_H > \frac{(1 - x_B)(p^H - u_L) + x_B(p^L - c_L)}{(1 - x_B)(u_H - u_L) + x_B(p^L - c_L)} = \bar{\alpha}_H \quad (28)$$

The Equilibrium

Proposition 1 *If $\alpha_H > \bar{\alpha}_H$ and $\delta > \tilde{\delta}$, the following profile of strategies constitutes an equilibrium:*

$$n_{SH} = \infty$$

$$n_{BL} = \infty$$

$$n_{BH} = 0$$

$$n_B = 0$$

$$A \text{ proportion } S_L^l(\delta) \text{ of sellers adopts } n_{SL} = +\infty$$

$$The \text{ remaining part chooses } n_{SL} = 0$$

4 Information Revelation and Welfare

Until now, we said nothing about the properties of the equilibrium presented in Proposition 1. Those properties are the object of this section. In the first subsection, we discuss information revelation. Since our result contradicts some assertions presented in Serrano and Yosha (2003), it is important to develop and neatly present our arguments. The second part is devoted to the welfare analysis.

Information Revelation

In the rational expectations literature the concept of information revelation is quite clear: prices reveal all available information about the state of the world. Even ignorant at the beginning, any agent is able to say what is the state of the world by observing the prices. The natural translation of that concept in our setup is that no trade takes place at a *wrong* price, i.e. at a price distinct from p_i at state $i = L, H$.

In our model, it is not possible that trade occurs at a wrong price when the state of the world is H . Indeed, at that state, sellers reject all prices which differ from p_H . We denote f_B the proportion of uninformed buyers who trade at a wrong price in state L . In case of complete information revelation, $f_B = 0$. Otherwise, f_B can be used to see if revelation of information is more or less complete.

Now, let us note that the proportion f_B is exactly equal to S_L^h , the proportion of sellers who play *tough*. This proportion tends to 1 when δ tends to 1. So, one can write

$$\lim_{\delta \rightarrow 1} f_B = 1 \quad (29)$$

It means that all uninformed buyers trade at a *wrong* price when the market becomes frictionless. One cannot imagine a worst situation in term of information revelation. Another important property is that the derivative of S_L^h with respect to δ is positive (at least for high value of δ). This property is naturally shared by f_B . Hence, increased frictions lead to a better information revelation.

The limit of f_B that we get is in contradiction with a remark in Serrano and Yosha (1993) which says that *Curiously, if trade in the limit does occur at the wrong price, the limiting fraction of wrong price trades is a precise number determined by [...]*

$$\frac{(1 - x_B) \frac{\alpha_H}{1 - \alpha_H} (u_H - p^H) - (p^L - c_L)}{(1 - x_B) [(p^H - u_L) - (p^L - c_L)]}$$

The number given by this expression is typically different from 1.

Actually, the validity of this assertion is limited to equilibria where n_B is not a corner solution of the uninformed buyers' problem. Indeed, for an intermediary step in Serrano and Yosha (1993)³, the equality $\frac{\partial}{\partial x} V_B = 0$ is used to characterize the optimal n_B . This equality is a first-order condition which is relevant only for interior solutions. Clearly in our case, the strategy for uninformed buyers is a corner strategy.⁴

Note that our condition (32) is exactly the condition that implies that the number given by Serrano and Yosha (2003) is larger than 1. Actually, it is not so surprising. It is indeed in that area that we would have suspected the existence of a corner solution for uninformed buyers. Intuitively, a lower number for n_B implies a larger number of trade occurring at the wrong price. When the number given in Serrano and Yosha (1993) tends (from below) to 1, it is right to think that n_B tends to zero. Once the zero is reached, n_B cannot decrease further and we expect indeed that, if it exists, the equilibrium with a corner strategy for uninformed buyers appears at that place.

Finally, Serrano and Yosha (1993) acknowledged the possibility of existence of some interior equilibria with incomplete revelation of information. However, our equilibrium is a corner one, and hence has not been stric

³The step 3 of the proof of proposition 2, see at p. 493

⁴It is maybe useful to underline that our new equilibrium does not invalidate Propositions 1 and 2 in Serrano and Yosha (1993). Only corollary following proposition 2 and some parts of proposition 3 have to be reevaluated.

sensu considered as possible by Serrano and Yosha (1993). Nevertheless, at the frontier, when the fraction here above is equal to 1, our equilibrium becomes interior. So, it is reasonable to believe that the equilibria which are not excluded by Serrano and Yosha (1993) really exist. However, it does not seem easy to get an explicit description of those equilibria. Moreover, we have no idea about the evolution of information revelation with respect to δ for those equilibria.

Welfare Analysis

A welfare analysis for this kind of model is not a novelty in the literature. Serrano and Yosha (1996) perform such an analysis for the one- and two-sided cases. Their main message is that information revelation and welfare go in the same direction. But their interest was limited to what happens when δ tends to 1. So, with them, we know that when δ tends to 1 we do not reach the *ideal* in term of welfare. But we do not know how to approach this *ideal* since Serrano and Yosha (1996) use a central planner to define this *ideal* but are silent on ways to implement this *ideal*. In what follows, we will study the evolution of expected utility, at the equilibrium established here above, for the different agents when δ varies. We discover that ex-ante, a decrease in δ profits to all the agents. Hence, we learn that the disappearance of frictions is not a desirable goal in all the equilibria.

Let us consider the forces affecting the welfare when the frictions vary. First, there is a direct effect. All the agents who do not trade immediately incur a cost of delay. This cost is larger when δ is lower. Then, there are the second-order effects. The level of friction affects the strategy of the agents. In our case, the only agents who vary their strategy are the sellers in state L . When δ decreases, sellers reduce their misrepresentation. The proportion of *tough* sellers goes in the same direction than δ . As we will see, this channel will create a positive impact of increasing frictions (decreasing δ) which overcomes the direct effect for all the agents. For the buyers, it is due to the fact that their opponents are readier to reach an agreement. For the sellers, it is due to a kind of externality.

State H In this state, our equilibrium implies immediate trading for all the agents. So, a modification of δ has neither a direct nor an indirect influence. So, to determine the benefit implied by a change in the level of frictions, one has to consider only what happens in state L . An appealing consequence is that a benevolent planner in charge of determining the optimal level of δ

will choose exactly the same value, irrespective from his knowledge or belief about the state of the world.

State L The results are less straightforward for this stage.

The least surprising result concerns the uninformed buyers. Those agents are not affected by the cost of delay since they trade instantaneously (indeed, they play always *soft*). The proportion of *tough* sellers evolves in the same direction than δ . When this proportion is lower, the probability to trade at the good price increases and thus also the expected payoff. Hence, an increase in the level of frictions (a lower δ) is a good thing for uninformed buyers since the incentives for the sellers to misrepresent is less important.

At first glance, the analysis for the sellers may seem quite complex. Actually it is not, at least if we remember that, at the equilibrium considered, the sellers are indifferent between all the strategies ($\Delta V_{SL} = 0$). As a consequence, we can study what happens for a seller who plays immediately soft and we will learn what happens for all the sellers. Thus, the expected payoff is given by

$$V_{SL} = B_L^l(p^L - c_L) + (1 - B_L^l)(p^M - c_L) = (p^M - c_L) - B_L^l(p^M - p^L) \quad (30)$$

A seller who plays always soft is not affected by the cost of delay since he reaches immediately an agreement. Unsurprisingly, this payoff is negatively affected by the proportion of *tough* buyers who do not accept a price as profitable for sellers as the one accepted by *soft* buyers. The proportion of *tough* buyers evolves in the same direction than δ . Hence, an increase in the level of frictions (a lower δ) is a good thing for sellers since it increases the proportion of *soft* buyers on the market.

It may be interesting to underline that if the proportion of *tough* buyers evolves with δ , it is due to the modification of the strategy of the sellers. When there are more *soft* sellers, the *tough* buyers reach more quickly an agreement and are thus faster to quit the market. Automatically, it increases the proportion of *soft* buyers which is a good thing for the sellers. So, there exists an externality between the sellers.

For informed buyers, there are two forces which go in opposite directions. The cost of delay increases with the frictions but the probability of disagreement (which is equal to (S_L^h)) decreases at the same time. It is not obvious which is the strongest force. The expected payoff is given by

$$V_{BL} = \sum_{i=0}^{\infty} (S_L^h \delta)^i (1 - S_L^h) (u_L - p^L) = \frac{1 - S_L^h}{1 - S_L^h \delta} (u_L - p^L) \quad (31)$$

We are interested by $\frac{dV_{BL}}{d\delta}$. It is sufficient to analyze the derivative of $\frac{1-S_L^h}{1-S_L^h\delta}$. We have

$$\frac{d\frac{1-S_L^h}{1-S_L^h\delta}}{d\delta} = \frac{S_L^h(1-S_L^h) - \frac{dS_L^h(\delta)}{d\delta}(1-\delta)}{(1-S_L^h\delta)^2} \quad (32)$$

This expression leads to an indeterminacy of the type $\frac{0}{0}$ when we are interested by its value at $\delta = 1$. We apply then l'Hospital's rule.

$$\left. \frac{d\frac{1-S_L^h}{1-S_L^h\delta}}{d\delta} \right|_{\delta=1} = \frac{\lim_{\delta \rightarrow 1} (1-S_L^h) \frac{dS_L^h(\delta)}{d\delta} - S_L^h \frac{dS_L^h(\delta)}{d\delta} - S_L^{h''}(1-\delta) + \frac{dS_L^h(\delta)}{d\delta}}{\lim_{\delta \rightarrow 1} 2(1-S_L^h\delta)(\frac{dS_L^h(\delta)}{d\delta}\delta + S_L^h)} \quad (33)$$

Once again, we get an indeterminacy of the type $\frac{0}{0}$ so we apply a second time l'Hospital's rule. After some simplifications, we get

$$\left. \frac{d\frac{1-S_L^h}{1-S_L^h\delta}}{d\delta} \right|_{\delta=1} = \frac{\lim_{\delta \rightarrow 1} \frac{d^2 S_L^h(\delta)}{d\delta^2} - 2(\frac{dS_L^h(\delta)}{d\delta})^2}{\lim_{\delta \rightarrow 1} 2(\frac{dS_L^h(\delta)}{d\delta} + 1)^2} \quad (34)$$

So, to have a negative derivative for V_{BL} , it is sufficient for $\frac{d^2 S_L^h(\delta)}{d\delta^2}$ to be negative. It is actually the case since

$$\frac{d^2 S_L^h(\delta)}{d\delta^2} = -\frac{2x_B(p^H - c_L)(p^L - c_L)(p^M - p^L)}{(1-x_B)(p^H - p^L)^3} < 0 \quad (35)$$

5 Equivalent Equilibrium in the Two-Sided Case

The results exposed so far already constitute an interesting contribution to the literature. Nevertheless, we would be happier if we had some evidence that the results are not limited to this model. In this section, we establish in the two-sided case as considered by Wolinsky (1990) the existence of an equilibrium which strongly resembles the equilibrium proposed in the previous sections.

This exercise is thus interesting, first by extending the validity of the intuition that frictions may improve the situation in term of information revelation and even in the sense of Pareto. This equilibrium then also leads us to reconsider the way noise force operates, thus improving our comprehension of the role played by the different forces at work in this kind of framework.

We will proceed in two steps. First we characterize (partially) the equilibrium. Then we prove that such an equilibrium indeed exists.

5.1 Partial Characterization

As in the one-sided case, an equilibrium is characterized by conditions which ensure that all the agents play a best response to the other actions and by conditions which ensure the stationarity. We will consider the second ones only at the end of this section. For the first conditions, we need to introduce the equivalent Δ 's which are :

$$\begin{aligned}
\Delta V_B &= \alpha_H(1 - S_H^h)(p^M - p^L) + (1 - \alpha_H)(1 - S_L^h)(p^M - p^L) \\
&\quad - \alpha_H(S_H^h)^2(p^H - p^M)\delta - (1 - \alpha_H)(S_L^h)^2(p^H - p^M)\delta \\
&\quad + \alpha_H S_H^h[\delta(u_H - p^M) - (u_H - p^H)] \\
&\quad + (1 - \alpha_H)S_L^h[\delta(u_L - p^M) - (u_L - p^H)] \\
\Delta V_S &= (1 - \alpha_H)(1 - B_L^l)(p^H - p^M) + \alpha_H(1 - B_H^l)(p^H - p^M) \\
&\quad - (1 - \alpha_H)(B_L^l)^2(p^M - p^L)\delta - \alpha_H(B_H^l)^2(p^M - p^L)\delta \\
&\quad + (1 - \alpha_H)B_L^l[\delta(p^M - c_L) - (p^L - c_L)] \\
&\quad + \alpha_H B_H^l[\delta(p^M - c_H) - (p^L - c_H)] \\
\Delta V_{BH} &= S_H^h[(-u_H + p^H - p^M + p^L) + \delta(u_H - p^M)] \\
&\quad - \delta S_H^h(p^H - p^M)] + (p^M - p^L) \\
\Delta V_{SL} &= B_L^l[(-p^H + p^M - p^L + c_L) + \delta(p^M - c_L)] \\
&\quad - \delta B_L^l(p^M - p^L)] + (p^H - p^M)
\end{aligned}$$

Those expressions are more complex than the ones used in the one-sided case. Nevertheless, the logic of those expressions is completely similar.

We do not write ΔV_{SH} and ΔV_{BL} because claim 1 is also valid in the two-sided case for informed sellers in state H and for informed buyers in state L . It is not the case for informed buyers in state H because we cannot exclude a priori the possibility to meet a *soft* seller in state H . Indeed, now there are some uninformed sellers.

What we call an equivalent equilibrium is an equilibrium where the agents present in the two models adopt qualitatively the same strategies. We would like that uninformed buyers play always *soft*. So, we will impose $\Delta V_B \leq 0$. Similarly for informed buyers in state H , we impose $\Delta V_{BH} \leq 0$. As in the one-sided case, we expect that informed sellers in state L are indifferent between all the strategies. Hence, we would like $\Delta V_{SL} = 0$. Since there was no uninformed sellers in the one-sided case, we do not know *a priori* the sign ΔV_S has to take. Nevertheless, if some sellers play *soft* in state L , it would create for informed buyers an incentive to play *tough*, at least for high δ . To avoid this case, we force ΔV_S to be equal or larger than 0.

To sum up, the wished equivalent equilibrium would be such that⁵

$$\Delta V_B \leq 0 \quad (36)$$

$$\Delta V_S \geq 0 \quad (37)$$

$$\Delta V_{BH} \leq 0 \quad (38)$$

$$\Delta V_{SL} = 0 \quad (39)$$

5.2 Existence

In this section, we will prove that conditions (36) to (39) can be simultaneously satisfied and we will give a profile of strategies which constitutes such an equilibrium.

The values of the ΔV_B and ΔV_{BH} imply that no buyer plays *tough* in state H . So, $B_H^l = 0$. As $\Delta V_S \geq 0$, $n_S > 0$ but since there is no *tough* buyers in state H , the uninformed sellers do not have the time to switch to a *soft* announcement. Indeed, they reach immediately an agreement with a *soft* buyer. Hence, $S_H^h = 1$. With this last value, we check easily that ΔV_{BH} is effectively lower than 0 for all $\delta < 1$ ⁶. Then, $n_{BH} = 0$ is compatible with an equilibrium.

Now, remark that ΔV_B is the same that the one-sided case when $S_H^h = 1$ and $\Delta V_{SL} = 0$ is identical in any case. So, S_L^h such that $\Delta V_{SL} = 0$ will imply the same B_L^l and the same value for ΔV_B .

For what follows, we need to be more precise in our notations. For uninformed sellers, we denote $\Delta V_S(x)$ the difference in expected gain between $n_S = x + 1$ and $n_S = x$. So, what we designated previously as ΔV_S is now designated as $\Delta V_S(0)$

Given $B_H^l = 0$, the update of the beliefs of uninformed sellers is perfect, i.e. in the second period on the market, they know that the state is L ($\alpha_H = 0$), since in state H there is no *tough* buyer. So, after one meeting with a *tough* buyer, $\Delta V_S(x) \forall x \geq 1$ is given by

$$\begin{aligned} & B_L^l [(-p^H + p^M - p^L + c_L) + \delta(p^M - c_L) - \delta B_L^l (p^M - p^L)] \\ & + (p^H - p^M) \\ = & \Delta V_{SL} \end{aligned} \quad (40)$$

⁵A very close equilibrium would be such that $\Delta V_B \geq 0$, $\Delta V_S \leq 0$, $\Delta V_{BH} = 0$ and $\Delta V_{SL} \leq 0$.

⁶ $\Delta V_{BH} = 0$ when $\delta = 1$.

So, it remains to check that $\Delta V_S(0) \geq 0$ for a newcomer and that will be sufficient to ensure the existence of this kind of equilibrium. For the newcomers, taking into account $B_H^l = 0$, one can write

$$\begin{aligned}\Delta V_S(0) &= (1 - \alpha_H)\Delta V_{SL} + \alpha_H(p^H - p^M) \\ &= \alpha_H(p^H - p^M) > 0\end{aligned}\tag{41}$$

The second equality is due to the fact that by construction of this equilibrium $\Delta V_{SL} = 0$. So, this equilibrium exists under the same conditions than his equivalent in the one-sided case.

Proposition 2 *If $\alpha_H > \bar{\alpha}_H$ and $\delta > \tilde{\delta}$, the following profile of strategies constitutes an equilibrium:*

$$\begin{aligned}n_{SH} &= \infty \\ n_{BL} &= \infty \\ n_{BH} &= 0 \\ n_B &= 0 \\ n_S \text{ in state } H &= \infty\end{aligned}$$

$$n_S \text{ in state } L \text{ and } n_{SL}^7 \text{ are such that } B_L^l = B_L^l(\delta).$$

It may be surprising that n_S depends on the state of the world but it is due to the fact that the learning is perfect for uninformed sellers.

5.3 Stationarity

We did not explicitly check that the equilibrium indeed satisfies the steady state conditions. In the previous subsection, we showed that

$$B_L^l = B_L^l(\delta)\tag{42}$$

$$B_H^l = 0\tag{43}$$

$$S_L^h = S_L^h(\delta)\tag{44}$$

$$S_H^h = 1\tag{45}$$

Hence, even if we do not write the conditions equivalent to (6), (7), (8) and (9), we know that the stationarity of these proportions are guaranteed. The conditions (4) and (5) are exactly the same for the two-sided case. It is easy to check that those conditions are satisfied without difficulties if $S_H^h B_H^l$ and $S_L^h B_L^l$ are different from 0. It is indeed the case for all $\delta < 1$.

⁷The proposition is presented as if n_S in state L and n_{SL} which ensure $B_L^l = B_L^l(\delta)$ are integers. If it is not the case, we can say as in subsection 3 that a part of agents plays *tough* during $+\infty$ periods while the remaining agents choose to immediately play *soft*.

6 Reconsidering the Noise Force

Gale (1987) was the first to suspect that the presence of uninformed agents on both sides of the market in Wolinsky (1990) was the source of the so negative result in terms of information revelation. Serrano and Yosha (1993) seemed to confirm the intuition of Gale (1987). The equilibrium that we discover proves that noise is actually not needed to get imperfect information revelation. This is not a real surprise since it was already proved by Blouin and Serrano (2001) in a model relatively similar.

What is more interesting for our understanding of the noise force is the equivalent equilibrium that we build for the two-sided case. Serrano and Yosha (1993) explained as follows the noise force : *As $\delta \rightarrow 1$ the informative content of the pairwise meetings decreases because there are more uninformed agents on both sides of the market trying to learn.* Actually, this vision of the noise force is not compatible with our equilibrium.

Indeed, in our equilibrium the unique agents who are uninformed are the uninformed newcomers. Among them, buyers will spend only one period on the market since they play always *soft*. Concerning the uninformed sellers, they remain uninformed only during their first period since one meeting is sufficient for them to learn with certainty the state of the world. The proportion of uninformed agents on the market in state L and H are respectively $\frac{M}{K_L}$ and $\frac{M}{K_H}$. The second proportion is constant in our equilibrium while $\frac{M}{K_L}$ is decreasing with δ . Indeed, the cost of misrepresenting decreases and implies an increase of misrepresentations by the sellers. In other words, more sellers are playing *tough* which induces more disagreements and finally more agents remain on the market, i.e. K_H increases with δ .

Hence, the noise force is not created by the fact that the proportion of informed agents decreases on the market when δ tends to 1. Indeed, this proportion evolves in the opposite direction. Nevertheless, the presence of uninformed agents on both sides of the market indeed creates a noise force. This noise is due to the fact that $n_S = +\infty$ and $n_B = +\infty$ are clearly not mutually compatible at a stationary equilibrium. If one or both strategies are different from $+\infty$ then the probability of trading at a wrong price is not nil. Remark that this argument is valid irrespective of the value of δ .

7 Conclusion

The main message of this work is : *Frictions are positive.* The result is very strong since, surprisingly, nobody is injured by an increase in the level of

frictions. Our result is established for the one-sided case but we show that an equivalent equilibrium exists in the two-sided case. So, the message is not specific to the one-sided case.

Nevertheless, we have to keep in mind the limits of the result; limits which, at the same time, indicate directions for further research.

First, our work is restricted to some equilibria. In the one-sided case for instance, if you consider an equilibrium with complete information revelation, frictions are harmful in terms of welfare. The properties are less obvious for the other equilibria in the two-sided case. It is really not impossible for a rise in the level of frictions to be a bad thing for everybody in some equilibria.

Second, we consider a particular model at least in two respects. The first particular feature is the assumption about the entry of new agents in the market. It would be interesting to treat the same issue in a model *à la* Blouin and Serrano (2001) which departs from Wolinsky (1990) and Serrano and Yosha (2003) concerning the entry of agents in the market. The kind of information ignored by the uninformed agents constitutes the second particular feature. In our context, the shadow information is a value common to all agents. A natural extension would be to consider adverse selection *à la* Akerlof. Results for $\delta \rightarrow 1$ are already provided by Blouin (2003).

Finally, what does a change in the level of frictions mean ? The best way of thinking is to consider that δ derives from an interest rate that we modify. There exists another interpretation but which is somewhat problematic. One could interpret a modification in the level of frictions as a change in the physical time between two matchings. The problem of this conception is that, actually, we modify at the same time the density per period of time of the flow of new agents in the market. Nevertheless, it must be possible to deal with this problem by defining M , the number of new entrants in the market, as a function of δ . It is quite difficult to guess what would change once this kind of modeling is adopted.

In addition to our main result, we get some very valuable byproducts. The proof of the existence of an equilibrium with incomplete revelation of information in the one-sided case is completely new. As discussed in section three, some other equilibria with incomplete revelation might exist. We produce also a reinterpretation of the noise which prevents complete information revelation in the two-sided case.

In a certain sense, we confirm an intuition present in the literature : *the pairwise meetings market is a procedure with bad revelation properties* (only in some equilibria of the steady-state version of the one-sided model

do some *good* equilibria show up). Gottardi and Serrano (2005) provide a general discussion of this and related issues.

Nevertheless, as shown by Isaac (2010), the existence of an equilibrium with complete information revelation is a robust result in the one-sided case even when we execute a dynamic analysis, i.e. without assuming a priori that the market is at a steady state. The result is actually stronger in Isaac (2010): under some conditions, there exists a *unique* equilibrium. Due to this uniqueness result, one could worry about the survival of our equilibrium in a dynamic analysis. First, the condition for the uniqueness in Isaac (2010) and the condition for the existence of our equilibrium are mutually exclusive. Moreover, the existence of our equilibrium in a dynamic analysis is almost proved in Isaac (2006). Only the last step is missing in Isaac (2006): the proof that the equilibrium exists when δ tends to 1. This step is a variation of our proof.

Moreover, in Gottardi and Serrano (2005) the intuition to explain the bad property of the matching procedure is that the matching process creates a kind of monopoly power. This monopoly power would be larger when the impatience of the agent increases. Hence, we expect that the situation is better when δ is higher. Actually, it is just the opposite in our result. So, in that perspective, our work tends to indicate that the link is quite more complex than expected between the modeling used in Gottardi and Serrano (2005) and the one used in the literature following Wolinsky (1990).

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